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GLOBAL ANOMALY AND UNDULATION RECOVERY USING GEOS-3 ALTIMETER DATA

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by

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The Ohio State University

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The Ohio State University Research Foundation

Abstract

This report describes the analysis of 3275 Geos-3 arcs of altimeter data containing 624670 frame averages. This data was adjusted to remove orbit error and altimeter bias in a primary adjustment and four regional adjustments. The root mean square crossover discrepancy was about ±55cm after the adjustment. The adjusted altimeter data, now considered to give geoid undulations, was used to predict values at 1° intersections from which an oceanic geoid map, with predicted accuracies, was prepared at a two meter contour interval. This geoid was compared to the GEM 9 geoid over very long profiles to examine the long wavelength error in the altimeter geoid. At a wavelength of 13010 km the root mean squares difference was 57cm. The altimeter geoid was also compared to altimeter geoids fixed by precise orbits. We found a root mean square difference of about 1 m with a systematic difference that implied the equatorial radius of the earth was 6378137 meters.

The adjusted altimeter data was also used to determine a total of 29479 1° x 1° anomalies (and undulations), 27466 of which had an accuracy of 15 mgals or better. Their average accuracy was 8 mgals. In addition, 957 5° mean anomaly and undulation values were computed. Representative anomaly differences with terrestrial estimates were 12 mgals for the 1° x 1° values and 7 mgals for the 5° values. Additional computations of point anomaly values were made to compare with ship data in the area of the Ninety East Ridge. There we found the altimetry anomalies followed quite well variations of the anomalies at 100 km wavelengths, and clearly showed correlation with bathymetry.

Foreword

This report was prepared by Richard H. Rapp, Professor, Department of Geodetic Science, The Ohio State University. This work was supported, in part, through NASA Grant No. NSG 5275, The Ohio State University Research Foundation Project No. 711118. The grant supporting this research is administered through the Goddard Space Flight Center, Greenbelt, Maryland with Dr. David Smith as Technical Officer.

Acknowledgment

The author is very indebted to a number of persons who helped in carrying out this research. Mr. Edward Herbrechtsmeier carried out the altimeter data handling, are merging, and the adjustment to remove orbit and altimeter bias. He also prepared a number of computer programs used in the study. Mr. Kostas Katsambalos carried out extensive programming support in arranging the predicted anomaly data and in other areas. Mr. Kamil Eren carried out a number of spectral analyses discussed in this study. Ms. Pamela Pozderac prepared the global altimeter geoid maps reproduced in the appendix from small maps made available to her. In addition Tony Watts provided ship gravity data and bathymetry for comparison with our altimetry computations.

For certain computations, computer funds were made available through the Department of Geodetic Science for use at the Instruction and Research Computer Center at The Ohio State University.

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Introduction

This report is an extension of the analysis with Geos-3 altimeter data described in Rapp (1977a). This extension was primarily oriented to processing an additional set of Geos-3 altimetry received during January and February 1978. As will be described in subsequent sections this new data was merged with the previous data and used to obtain a near global oceanic geoid map and a large number of 1°x 1° mean anomalies. In addition a number of computations involving point gravity anomalies was carried out for comparisons with actual ship measurements.

The Data Set

The data used in our investigations is the intensive mode data where we use frame averages. (A frame average represents a mean over 2.048 seconds or 3.277 seconds.) In taking the original data supplied by NASA (through the Wallops Flight Center), an editing procedure is used that has six different edit criteria. These criteria are designed to remove most of the bad or poor altimetry data. In our first analysis (ibid, 1977a), we accepted 1976 arcs containing 419,294 frame measurements. The location of this edited data is shown in Figure 1.

During January and February 1978 a set of approximately 70 tapes were received containing additional Geos-3 altimeter data. The location of this unedited data is shown in Figure 2.

Portions of the data were arcs that were just the continuation of previous arc segments. A procedure was developed to merge common, continuous arcs, of the old and new data sets whenever possible. After editing and merging, the altimeter data set available for further processing is shown in Figure 3. This data set now contained 3275 Geos-3 arcs with about 624670 frame averages. The same data set is shown in Figure 4 which also shows the location and numbering of the 5° equal area anomalies used in this study.

Bias Removal and the Adjustment Process

The altimeter data that is available in the edited form is subject to errors caused by a bias in the altimeter and orbit errors. These errors must be removed (as much as possible) before additional processing is carried out. To do this a procedure was described in Rummel and Rapp (1977) and Rapp (1977a) that combined a fit of the altimeter implied geoid undulations to a satellite derived geoid with crossover constraints to obtain parameters of an error polynomial. After these parameters are found for each arc, adjusted geoid heights can be found. (We note here that formally we are dealing with sea surface heights. If we neglect sea surface topography geoid heights would be found.)



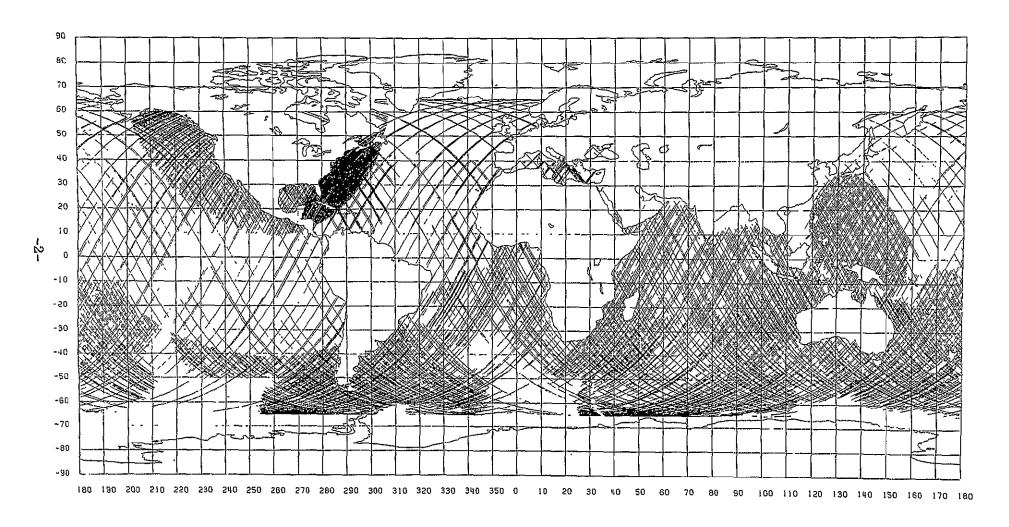


Figure 1. Location of Edited Arcs in First Altimeter Analysis (Rapp, 1977a).

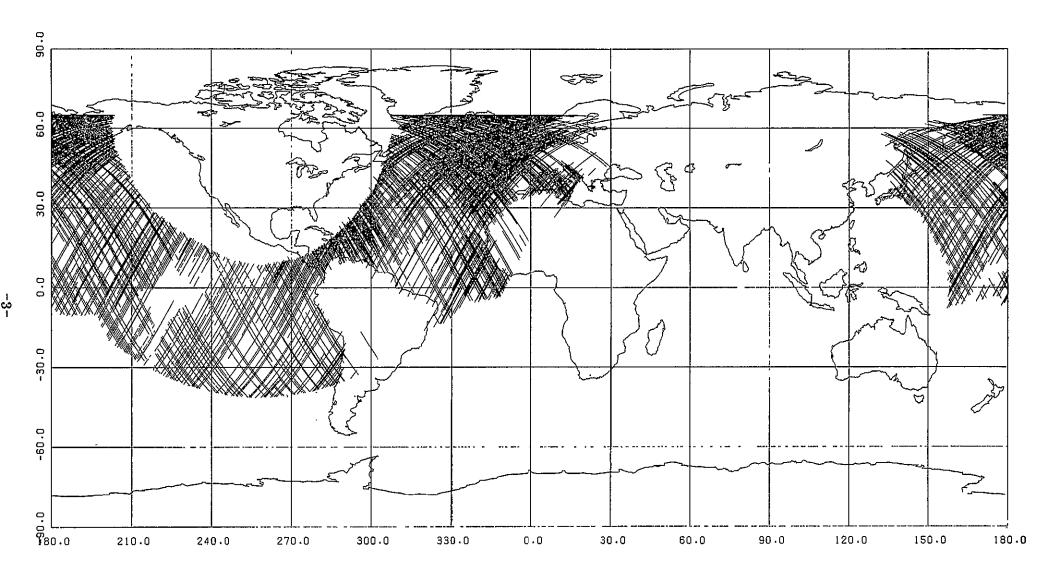


Figure 2. Location of Additional Unedited Altimeter Arcs.

GEOS 3 GROUND TRACKS

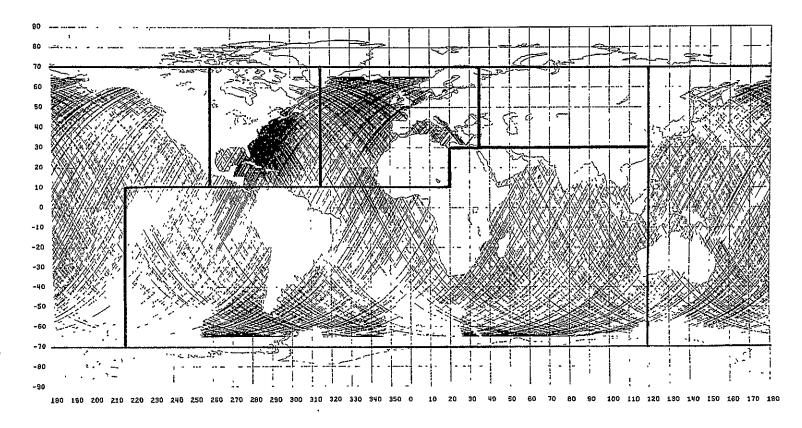


Figure 3. Location of Edited Geos-3 Altimeter Data and Areas of Regional Adjustments.



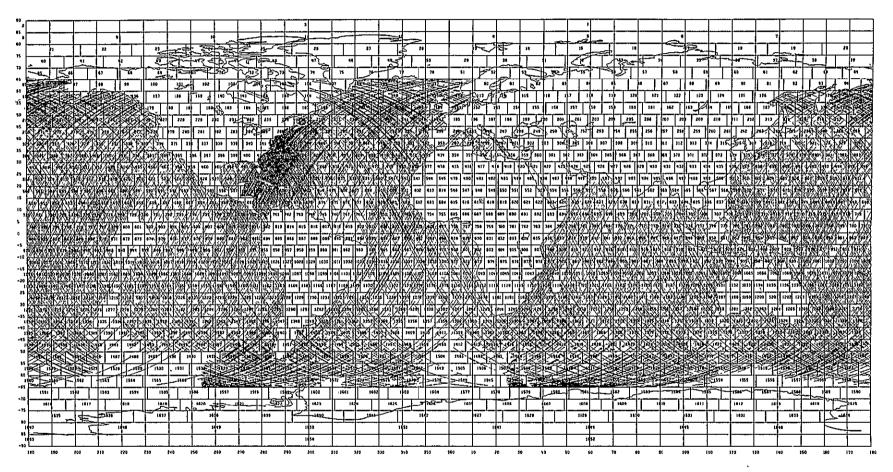


Figure 4. Location of Edited Geos-3 Arcs and 5° Equal Area Blocks.

For this work a procedure almost identical to that described in the earlier work (ibid, 1977a) was carried out. We first made an adjustment of a primary network of altimeter arcs that were chosen for their arc length and representative global distribution. After these arcs were adjusted, four regional adjustments were performed enforcing the primary arcs in the region.

The mathematics of the adjustment process are described in detail in Rapp (1977a) as well as the procedures for determining the crossover locations. The reference field used for the reference geoid was the GEM 9 (Lerch et al. 1977) set of potential coefficients taken to degree 20. For arcs whose length were less than 18° (305 seconds) only a single bias term was solved for. For the longer arcs a bias term and a trend term were found.

The location of the regional adjustments are given in Table 1 and shown in Figure 3.

| Name | Geographic Limits | | | |
|-----------------|------------------------|--------------------------|--|--|
| | φ [¢] | λ° | | |
| New Calibration | 70 to 10 | 260 to 314 | | |
| East Atlantic | 70 to 10 | 314 to 34 | | |
| Africa – India | { 30 to 10 10 to 70 | 20 to 120 218 to 120 | | |
| Pacific | 70 to-70 70 to 10 | 120 to 218 218 to 260 | | |

Table 1. Regional Adjustment Locations

Information on the adjustment of the primary and regional networks is given in Table 2.

Table 2. Adjustment Statistics Related to the Primary and Regional Adjustments.

| | | Numb | er of | Crossover Discrepancies | | |
|--|---------------------------------|-------------------------------------|---|--|---|--|
| Region | Arcs | Unknowns | Obser- vations | Cross- overs | apriori | a posteriori |
| Primary New Calibration East Atlantic Africa - India Pacific | 700 478 273 588 408 | 1383 800 . 464 1074 707 | 263077 81822 47065 139772 95947 | 10149 35403 9144 15977 12041 | ± 7.81 m 6.94 8.56 7.42 10.38 | ± 0.60 m 0.47 0.56 0.50 0.60 |

We should note here that the adjustment process is carried out in two stages. A preliminary adjustment is made and adjusted crossover discrepancies are examined. Cross points having discrepancies greater than about 3.5 meters are deleted. In some cases a whole arc segment may be deleted. The adjustment is then repeated a second time to obtain the data given in Table 2.

The crossover discrepancies after the adjustment, as shown in Table 2, are of the same magnitude as found in the first adjustment (Rapp, 1977a, Table 10).

The adjusted undulations from this new adjustment (with additional data) were compared to the corresponding values from the earlier adjustment. The results of these comparisons are given in Table 3.

Table 3. Comparison of Adjusted Point Geoid Undulation of New and Old (Rapp, 1977a) Adjustments.

| Area | Number of | Mean | RMS | Maximum RMS |
|-------------|-----------|------------|-----------------------|-------------|
| | Arcs | Difference | Difference | Difference* |
| Primary | 337 | 0.06 m | $\pm 0.33 \mathrm{m}$ | ±2.04 m |
| Calibration | 154 | -0.09 m | $\pm 0.29 \mathrm{m}$ | ±1.78 m |

^{*} Along any one arc.

We see that on the whole the differences between the adjustments is on the order of 30 cm (RMS) although over a few arcs the differences may reach the 2 meter level.

The Altimeter Geoid

Disregarding sea surface topography effects, we may regard the adjusted altimeter data to give us geoid undulations with respect to an ellipsoid of defined flattening (1/298.256) but whose equatorial radius is specifically undefined. It's conceptual definition is, however, the equatorial radius of the ellipsoid for which the global mean geoid undulation is zero.

A global oceanic geoid (or mean sea surface) has been computed from the adjusted data using primarily the procedure described by Kearsley (1977). This procedure predicts a geoid undulation (and its accuracy) at grid intersections from the surrounding point altimeter undulations. The predictions were made using least squares prediction techniques using covariances from subroutine COVA (Tscherning and Rapp, 1974) and including the data noise (with a ± 0.4 m contribution from the effect of errors in the GEM 9 potential coefficients on the adjusted geoid) in the process. The grid interval chosen was 1° x 1° using the 5 closest altimeter points. Choosing the grid interval this large can result in

the loss of detailed features below the scale of 100 km. However, the cost of computing a global oceanic geoid at a finer scale is prohibitive for us.

The grid values were contoured using a new contouring program developed by Sünkel (1979) that uses spline functions to obtain smoother contours than seen in Kearsley (1977). The computations were made, roughly, in 30° x 30° blocks. All grid undulations and their accuracies were printed out and are available for other use. The result of the contouring (for the undulations and the accuracies) were output from a small (11" wide) Versatec plotter. The individual blocks were fitted together to produce four map sheets (undulation and accuracy maps for the Eastern and Western Hemispheres) whose original size was approximately 39" x 28". Reduced versions of these maps are shown in Figures 5, 6, 7, and 8. Almost full size prints of these maps have also been made. Areas that are blank on these maps did not have sufficient altimeter data for preparing a geoid plot.

No attempt here is made to interpret this gooid. The features obvious from potential coefficient geoids is present as are the details from short wavelength geophysical structures.

Geoid Comparisons

It is of interest to compare the gooid given in Figures 5 and 6 (or more precisely the predicted grid values) with other sources. In the past comparisons with the altimeter geoid and some other reference geoid have been made over the relatively short arc segments of Geos-3 data. Our first interest here is to compare the GEM 9 undulations (to degree 20) to the altimeter derived undulations in long profiles having a constant latitude or a constant longitude. For a number of different tests 7 long (~14000 km) profiles were chosen. A plot of the GEM 9 geoid and the adjusted altimeter geoid for two typical profiles (two and five) are shown in Figures 9 and 10. (The origin for profile two is at $\lambda = 10^{\circ}$, while the origin for profile five is at $\varphi = -65^{\circ}$.) Information on these profiles is given in Table 4.

Profile: Mean Diff. RMS Diff. Longitude Latitude Length -48° 10° to 186° 14732 km $-0.6\,\mathrm{m}$ $\pm 2.3 \,\mathrm{m}$ Two -65° to 52° 335° 14455 km $0.7\,\mathrm{m}$ $\pm 2.6 \,\mathrm{m}$ Five

Table 4. Comparison of GEM 9 and Altimeter Geoid Profiles.

These plots do not reveal any significant systematic differences between the two undulation sets.

The undulation differences (GEM 9 minus altimeter derived) were analyzed by Eren (1979, private communication) to determine their power spectrum. The

Figure 5.

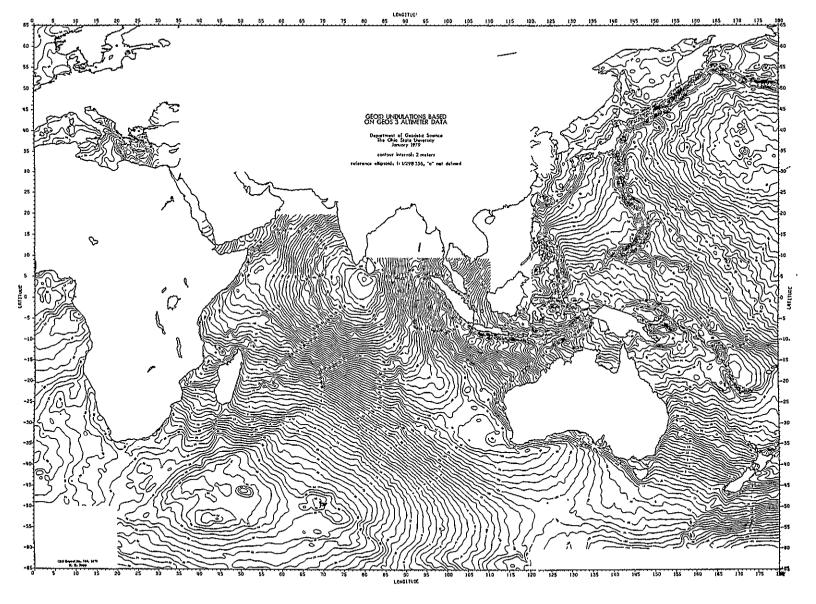


Figure 6.

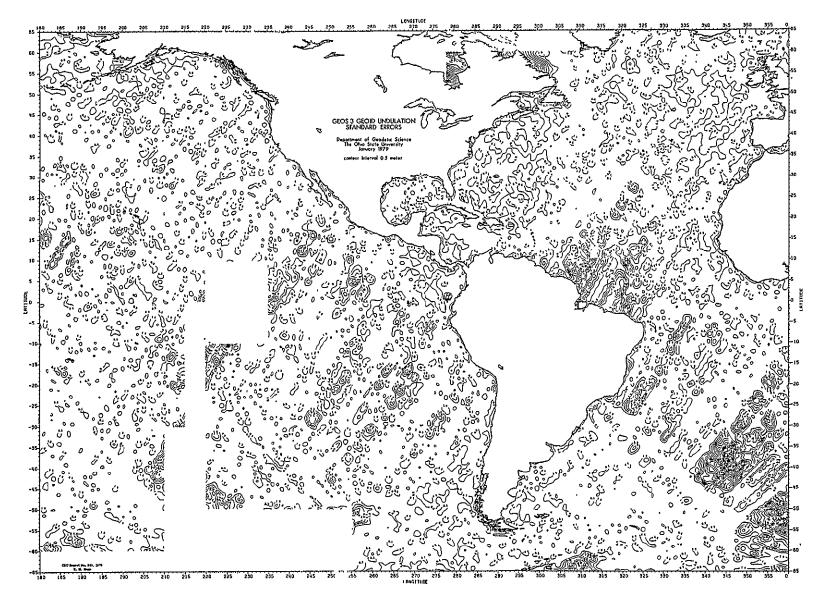
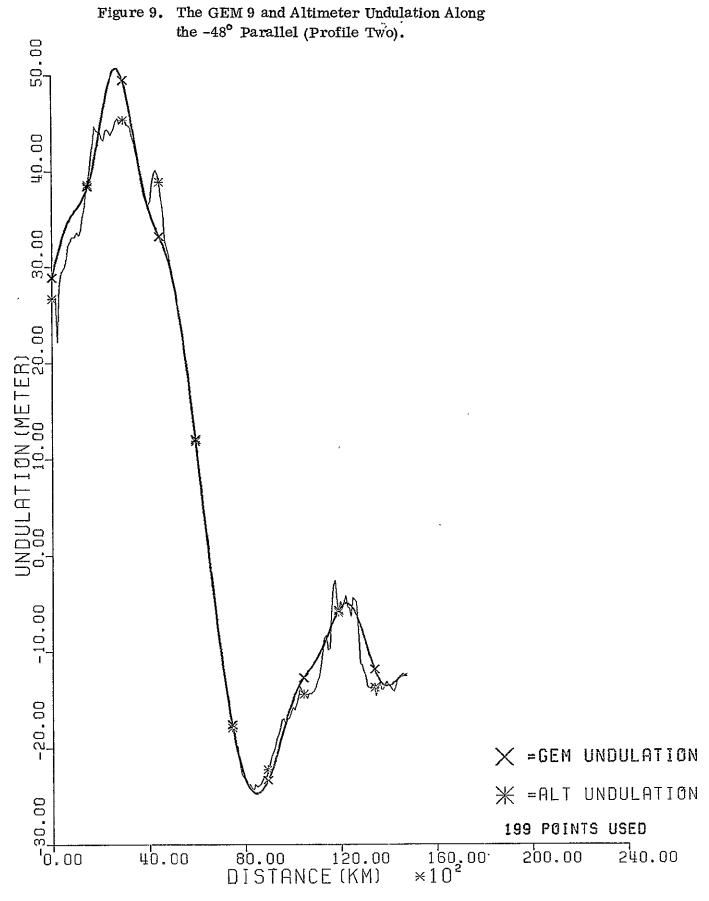
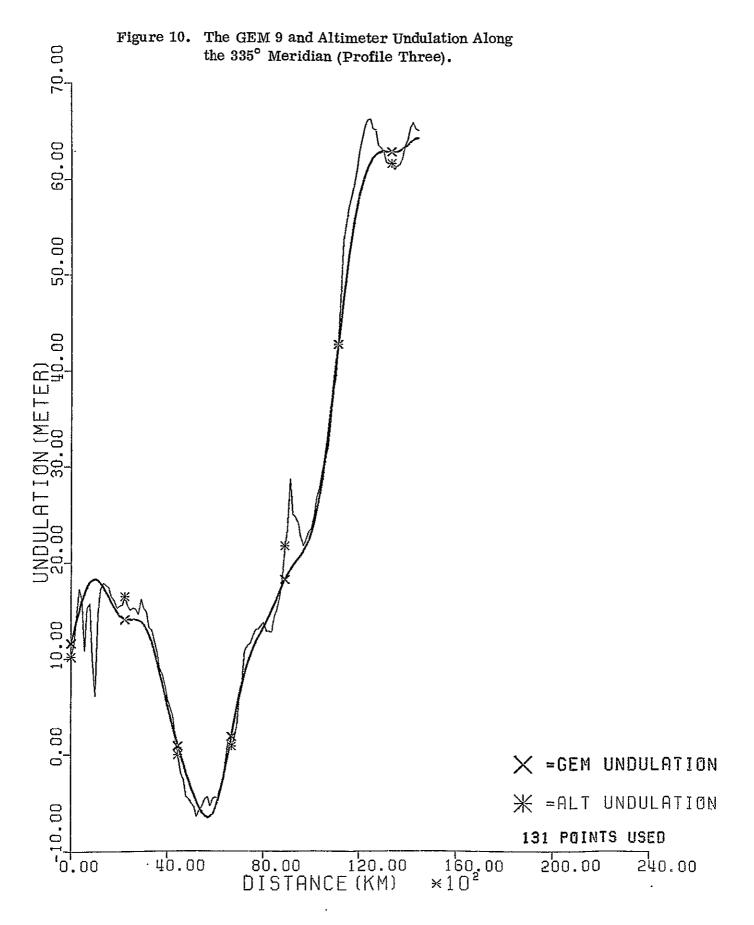


Figure 7.

Figure 8.



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power, by wavelength is given in Table 5 based on averaging the results for the 7 individual profiles.

| Wavelength (km) | Approx. S.H. Degree | Power (m²) | (Power) ^{1/2} (m) | GEM 9 Error (m) |
|-------------------------|---------------------|------------|----------------------------|--------------------|
| 13010 | 3 | .33 | ±.57 | ±.04 |
| 6505 | 6 | .61 | .78 | .16 |
| 4337 | 9 | .60 | .77 | .38 |
| 3253 | 12 | .58 | .76 | .43 |
| 2602 | 15 | . 49 | .70 | .51 |
| 2168 | 19 | .15 | .39 | .53 |
| Total to S.H. Degree 19 | | 2.76 | 1.7 | 1.7 m* |

Table 5. GEM 9 Minus Altimeter Geoids by Wavelength.

The above computation assumed that the data was regarded as periodic. An alternate procedure assuming non-periodic data, with a window function was also carried out with similar results.

From Table 5 we see that at long wavelengths we can expect errors in our altimeter good up to about 0.75 meters. At shorter wavelengths (2168 km) the difference is similar to the expected error.

Additional computations could be carried out in this area by using more profiles and introducing data noise.

Our altimeter geoid was also compared to altimeter geoids presented by Brace (1977) and by Marsh et al (1978). The Marsh et al geoid (or mean sea surface) is based on the computation of certain laser reference orbits which are used to obtain reference altimeter undulations. These latter undulations are then used as a frame for a crossing arc adjustment process. The comparisons were made for the three areas in Marsh et al where the plotted contour interval was 1 meter. The values at grid intersections were read from the contour maps of Marsh, and taken as the predicted values from our data. The results are given in Table 6.

Mean Diff. Author Area Auth. - Rapp

Table 6. Geoid Comparison from Various Sources.

^{*} Due to errors in the GEM9 potential coefficients from n = 2 to n = 19.

The Brace undulations are based on Doppler satellite orbits and on a crossing arc adjustment process.

The mean differences can be due to equatorial radius used as a reference (6378140 m for Marsh, and 6378135 m for Brace) and/or an uncorrected bias term. We will assume the bias has been correctly considered in Marsh while a correction of 2.5 m is to be subtracted from the Brace data (Anderle, 1979). The resultant mean differences then will imply an ideal equatorial radius as given in Table 7.

Table 7. Equatorial Radius Implied by the Altimeter Geoid Comparisons.

| Comparison with: | Equatorial Radius |
|------------------|-------------------|
| Marsh et al | 6378137.1 m |
| Brace | 6378137.1 m |

The agreement of the two values is fortuitous. We must remember, however, that these values assume the orbit scale is correct. A formal accuracy assessment was not carried out but ± 2 meters seems reasonable.

The Computation of Mean Covariance Functions

The estimation of the mean anomalies and mean undulations was to be done by the method of least squares collocation using the procedures described in Rapp (1977a). As part of these procedures it is necessary to determine point and mean covariance functions. As originally used in Rapp (ibid) the mean covariance functions were determined from the numerical integration of point covariance functions. An alternative method, used earlier (Tscherning and Rapp, 1974), by Schwarz (1976) and others, is to introduce the smoothing operator for degree ℓ , β_{ℓ} (Meissl, 1971) into the series expressions for the needed covariance functions. The mean covariances needed were cov ($\Delta g, \Delta g$), cov ($N, \Delta g$), and cov (N, N) where an unbarred quantity represents a point value and a barred quantity represents a mean value. In our test computations we shall restrict the mean value to a 1°x 1° anomaly block

A point anomaly covariance function, given with respect to a reference field of potential coefficients to degree ℓ (ref) can be computed from (Tscherning and Rapp, 1974):

$$cov (\Delta g, \Delta g) = \sum_{\ell=\ell}^{\infty} c_{\ell} s^{\ell+2} P_{\ell} (\cos \psi)$$

$$\vdots \quad \ell = \ell (ref) + 1$$
(1)

where c_{ℓ} are anomaly degree variances and $s = (R_8/R)^2$ where R_8 is the radius of the Bjerhammer sphere (internal to the earth) and R is the mean radius of the earth. One estimate for s is 0.999617 (Tscherning and Rapp, 1974). To obtain the mean covariances we introduce β_{ℓ} :

$$\operatorname{cov}(\overline{\Delta}g, \overline{\Delta}g) = \sum_{\ell=\ell}^{\infty} c_{\ell} \beta_{\ell}^{2} s^{\ell+2} P_{\ell}(\cos \psi)$$
 (2)

The smoothing operator β_{k} can be computed from the following (Rapp, 1977b)

$$\beta_{\ell} = \cot \frac{\psi_0}{2} \quad \frac{P_{\ell 1}(\cos \psi_0)}{\ell (\ell + 1)} \tag{3}$$

where ψ_0 is the radius of a cap that has the same area as the block (e.g. 1° x 1°) being considered. As we are dealing with 1° x 1° values the areas will be latitude dependent so that β_{ℓ} , in our application, will be latitude dependent.

The other covariances needed can be found by the law of propagation of covariances (Moritz, 1972, 1978). In spherical approximation we have:

$$\operatorname{cov}(N, \overline{\Delta}g) = (R/G) \sum_{\ell=\ell}^{\infty} c_{\ell}/(\ell-1) \beta_{\ell} s^{\ell+1} P_{\ell}(\cos \psi) \qquad (4)$$

$$\operatorname{cov}(N, \overline{N}) = (R/G)^{2} \sum_{\ell=\ell}^{\infty} c_{\ell}/(\ell-1)^{2} \beta_{\ell} s^{\ell+1} P_{\ell}(\cos \psi)$$
 (5)

where G is an average value of gravity. In actual computations the summation to $^{\circ}$ was replaced by a summation to degree 540 which is sufficiently high so that no significant change will take place by going to a higher degree. Tables of these mean covariances were constructed at 0.05 intervals. The original program was modified by removing the numerical integration procedure for the 1° x 1° block computations and replacing it with table interpolation procedures.

These methods were tested by carrying out predictions in two 5° equal area anomaly blocks both of which had a north latitude of -10° . The series covariance functions were computed with a β value for $1^{\circ} \times 1^{\circ}$ blocks at the equator. Predictions were carried out using the numerical integration procedure and the directly tabulated mean covariance functions. The differences in the $1^{\circ} \times 1^{\circ}$ mean anomaly predictions was about ± 0.6 mgals while the mean undulation differences were on the order of $\pm .1 \, \mathrm{m}$. The estimated standard deviations were essentially unchanged. We thus conclude that this procedure could be applied successfully.

The computer time savings were clearly seen in a run where the error covariance matrix (see later discussion) was also computed. The savings was 14 secs (on an IBM 370/168) or approximately 8% of the total time. This is a representative value only as the specific savings will depend on the number of given data points (alternate undulations).

Although the savings could be significant over repeated computations, the decision was made <u>not</u> to implement these procedures in our operational runs. One reason for this is that we felt that we would have to compute new covariances for different latitude areas because of the change in β 's with the area of the block. (For example, for a 1°x 1° at the equator $\beta = 0.564$ while at latitude

 60° , $\beta = 0^{\circ}.402$.) It could be that sufficient accuracy could be found by adopting some average β values but this was not tried. Additional testing in this area is needed as well as the testing of series expression for the 5° equal area predictions that were also carried out.

The Error Covariance Matrix

In the computations performed previously (Rapp, 1977a) for mean anomalies and undulations, the 1°x 1° values within a 5° equal area block were predicted from the same data set selected in and around the 5° block. This procedure had the advantage that only one matrix inversion was required; it did however have the disadvantage that the predictions of the 1°x 1° blocks could be highly correlated. To specifically consider this question we considered the following expression for the error-covariance matrix for the least squares collocation procedure (Moritz, 1972):

$$E_{ss} = C_{ss} - C_{sx} \overline{C}^{-1} C_{xs}$$
 (6)

where: Ess is the error covariance matrix of the signals being predicted;

Css is the (physical) covariance matrix of the signals being predicted;

 C_{sx} is the cross covariance matrix between the signals being predicted and the observations:

 \overline{C} is the covariance matrix of the observations plus the noise covariance matrix;

 C_{xs} is C_{sx}^{\dagger} .

In our case the observations are the altimeter derived undulations while the signals being predicted are the $1^{\circ} \times 1^{\circ}$ mean gravity anomalies (or undulations). The size of the E_{ss} matrix will depend on the number of $1^{\circ} \times 1^{\circ}$ blocks within the 5° equal area block. Near the equator E_{ss} would be a 25 x 25 matrix while at latitude 65° E_{ss} would be a 55 x 55 matrix. The diagonal elements of E_{ss} would be the square of the predicted accuracy of the individual blocks. E_{ss} can also be converted into a correlation coefficient matrix.

Equation (6) was evaluated for several cases of possible interest. In the first case only one known undulation value was used to estimate all the $1^{\circ} \times 1^{\circ}$ anomalies in a 5° block. The maximum correlation found was 0.53. When 88 data points were used in this same 5° block, the maximum correlation coefficient was now .13.

Two additional 5° blocks were also investigated. One 5° block (at $\varphi_n = -12^{\circ}.5$) had 391 data points used for the prediction. This would be considered a block with fairly dense altimeter data. In this case the largest correlation coefficient was 0.20. A second block, in the same latitude, having a less dense data set (261 points), was also considered. In this case the largest correlation coefficient reached 0.34.

We thus see that with increasing data sets the correlation between the blocks decreases. For most computations we will be doing, we would expect the correlation coefficient to be 0.3 or less. Because of this small magnitude we can regard most anomalies as almost independently determined. We therefore choose not to compute the Ess matrix for our operational work.

Reference Model Error

The prediction of mean gravity anomalies and geoid undulations has been carried out with respect to a reference gravitational field model defined by the GEM 9 potential coefficients (Letch et al., 1977) taken to degree 20. Specifically the anomaly and its estimated accuracy is computed from the following equations (Rapp, 1977a, 1978a):

$$\Delta g = \underline{C}_{ghR} \left(\underline{C}_{hhR} + \underline{D} \right)^{-1} \left(\underline{h} - \underline{h}_{R} \right) + \Delta g_{R}$$
 (7)

$$m_g^2 = \underline{C}_{ggR} - \underline{C}_{ghR} \left(\underline{C}_{hhR} + \underline{D}\right)^{-1} \underline{C}_{hgR}$$
 (8)

where Δg is the predicted free-air gravity anomaly with respect to ellipsoidal gravity field;

h is a column vector of the altimeter implied geoid undulations;

C_{ghR} is the row vector containing the covariance (referred to the reference field) between the anomaly being predicted and the given geoid undulation;

 C_{hhR} is the square, symmetric matrix containing the covariances (referred to the reference field) between the given geoid undulations. If there are n + b values being used this matrix is $n \times n$;

D is the error-covariance matrix of the given geoid undulations which was taken to be a diagonal matrix whose elements corresponded to the square of the standard deviation of the altimeter measurement;

C_{ggR} the expected mean square value (referred to the reference field) in a global sample, of the anomaly being predicted;

m, the predicted standard deviation of the predicted anomaly;

 Δg_R , the gravity anomaly and geoid undulation implied by the reference N_R , h_R set of potential coefficients.

Similar equations can be written for the estimation of the mean undulations.

In our previous computations no consideration was given to the error in the final result caused by errors in the GEM 9 coefficients. One way suggested by Colombo (private communication, 1979) is to add to the covariance matrices, given with respect to the reference field, a component implied by the errors in the potential coefficients. This component can be obtained from equation (2), (4), and (5) by setting $\beta_{\ell} = 1$ and replacing the c_{ℓ} values by the errors in the anomaly degree variances implied by the errors in the potential coefficients.

Such a computation was done and prediction results for the $1^{\circ}x\ 1^{\circ}$ anomalies and undulations in a 5° block were compared. We found changes in the predicted $1^{\circ}x\ 1^{\circ}$ anomalies of only about ± 0.2 mgals with the standard deviations increasing about 0.3 mgal (from about ± 7 mgal). The undulation values and their accuracy changed no greater than 0.2 meter.

We thus conclude this the errors in the GEM 9 coefficients, when used as a reference field, do not significantly contribute, to our final error estimate, provided the above procedure is correct.

Anomaly Differences Implied by the New and Old Adjustments

In our previous report we had computed 9995 $1^{\circ}x 1^{\circ}$ anomalies. By February 1978 this number had increased to 12144 values, all values being based on the adjusted undulations from our first adjustment process. After completing the second adjustment of the altimeter data we computed a large number of $1^{\circ}x 1^{\circ}$ anomalies to compare with our earlier estimates. Some statistics on these comparisons are shown in Table 8.

| Table 8. | Comparison of 1° x 1° Mean Anomalies Computed from New |
|----------|--|
| | and Old Adjustments of the Altimeter Data. |

| Area* | Number | Mean Diff. | RMS Diff. (mgals) | Max. Diff. (mgals) |
|---------------|--------|------------|----------------------|-----------------------|
| Calibration | 489 | 0.2 | ±4.9 | 34.3 |
| South America | 1675 | 0.2 | ±2.3 | 17.2 |

^{*} see Rapp (1977a)

The root mean square differences are all smaller than the predicted accuracy of the $1^{\circ} \times 1^{\circ}$ anomalies and thus such differences are not of great concern. There is some concern over a few large discrepancies (such as the 34 mgal difference). The reason for this is not clear although it may be related to a singly bad data point.

The anomalies from the new and old adjustment were also compared to terrestrial anomalies. These comparisons are shown in Table 9.

Table 9. Comparison of 1°x 1° Mean Anomalies from the New and Old Adjustment to Terrestrial Data.

| | Old Ad | justment | New Ad | justment |
|--|--------------------------|------------------------------------|--------------------------|---------------------------------------|
| Area | Number | RMS Diff. | Number | RMS Diff. |
| Calibration South America Alaska Phillippines | 260 382 492 527 | ± 8.9 mgals 16.5 9.5 12.5 | 265 491 524 194 | ±10.7 mgals 17.7 10.5 - 12.4 |

It seems clear that the old adjustment yields slightly better anomaly values. Again considering the predicted anomaly accuracies these differences are not significant. However they are sufficient to have us accept in our final results, anomalies predicted from the old adjustment where such data exists in sufficient quantity to obtain reliable predictions.

Effect of the Mass of the Atmosphere

The computations for the gravity anomalies made through equation (7) have made no assumption on the attraction of the mass of the atmosphere. For points internal to a spherical shell comprising the atmosphere the attraction of the atmosphere is zero. However in gravity anomaly computations the current procedure is to include the mass of the atmosphere within the mass of the reference ellipsoid. Thus for comparisons of altimeter derived anomalies with terrestrial anomalies derived using a gravity formula based on a reference ellipsoid that contains the mass of the atmosphere, a small correction to the altimeter anomaly is needed. This correction can be found by first defining a gravity anomaly with respect to an ellipsoid for which the mass of the atmosphere is not included:

$$\Delta g_{E} = g - \gamma_{E} \tag{9}$$

where E indicates the earth mass, g is observed gravity (properly reduced) and γ_{ϵ} is normal gravity. The corresponding anomaly when the mass of the atmosphere is included in the reference ellipsoid is:

$$\Delta g_{E_{+A}} = g - \gamma_{E_{+A}} \tag{10}$$

Thus:

$$\Delta g_{E+A} = \Delta g_E - (\gamma_{E+A} - \dot{\gamma}_E)$$
 (11)

For points (or blocks) located at a zero elevation $(\gamma_{E+A} - \gamma_E)$ is 0.87 mgals. (IAG, 1971). Thus, before comparing the altimeter derived anomalies to terrestrial anomalies that would be referred to the gravity formula of the Geodetic Reference System 1967, (for example) 0.87 mgals should be subtracted from the altimeter derived anomaly.

Effect of Systematic Undulation Error on the Anomaly Predictions

The "geoid undulations" used in our estimation process may have systematic errors caused by the neglect of sea surface topography with respect to the geoid. To see the effect we carried out a computation of the 1°x 1° anomalies and undulation in a 5° equal area block using altimeter derived undulations from which a constant 1 meter had been removed. Comparison with the same predictions without the systematic change revealed a systematic difference in the anomaly predictions of 2.5 mgals. This change is below the average accuracy of the predicted anomalies by a factor of 3.

Subsequent comparisons of the 1°x 1° predicted anomalies to terrestrial anomalies indicates a mean difference of 0.5 mgals. This difference could be due to sea surface topography effects that do not average to zero, to an error in the equatorial gravity of the Geodetic Reference System 1967, or it may be statistically insignificant.

Effect of Covariance Functions Used on the Predicted Anomalies and Undulations

The predictions defined by equation (7) require certain covariance functions. In Rapp (1977a) the covariance functions were obtained from subroutine COVA (Tscherning and Rapp, 1974) with respect to a degree 20 field based on a certain anomaly degree variance model. The covariances used are called global covariances in the sense that they are based on parameters representative of the global gravity field. One argument is that it would be more reasonable to use covariance functions specifically designed for a given area. It thus seems appropriate to consider the sensitivity of our predictions to the covariances used in the prediction process.

To do this we choose to work with three different covariance functions carrying out predictions in 1°x 1° blocks within several 5° equal area blocks. The first covariance function group was that implied by the anomaly degree variance model described in Tscherning and Rapp (1974). This model was used to compute the needed covariances with respect to a degree 20 reference field. These covariances were the ones used in all of our previous computations (Rapp, 1977a).

The second covariance function was based on an anomaly degree variance model developed by Jekeli (1978) and designated as the 1 L model. This model used the same form of the anomaly degree variance model as used by Tscherning and Rapp but computed the model parameters enforcing a low horizontal anomaly variance (C_0) considerably less than the T/R model, and an anomaly correlation length ξ (i.e. $C(\xi) = C_0/2$) that was about twice that of the T/R model.

The third covariance function group considered was that described by Jordan (1972) for local use. Jordan discussed a third-order Markov undulation covariance

model, from which an anomaly covariance model, and an anomaly-undulation cross covariance model could be derived. Specifically we have:

$$C(N, N) = C_0(N) \left(1 + \frac{r}{D} + \frac{r^2}{3D^2}\right) e^{-r/D}$$
 (12)

$$C (\Delta g, \Delta g) = C_0(\Delta g) \left(1 + \frac{r}{D} - \frac{r^2}{2D^2} \right) e^{-r/D}$$
(13)

$$C (\Delta g, N) = \frac{2\sqrt{C_{0}(N)C_{0}(\Delta g)}}{\sqrt{6}} \left[\frac{\mathbf{r}}{2D} \left(1 - \frac{\mathbf{r}^{2}}{2D^{2}} \right) \left[I_{0} \left(\frac{\mathbf{r}}{2D} \right) K_{1} \left(\frac{\mathbf{r}}{D} \right) - I_{1} \left(\frac{\mathbf{r}}{2D} \right) K_{0} \left(\frac{\mathbf{r}}{2D} \right) \right] + \frac{\mathbf{r}^{2}}{4D^{2}} \left[I_{0} \left(\frac{\mathbf{r}}{2D} \right) K_{0} \left(\frac{\mathbf{r}}{2D} \right) + I_{1} \left(\frac{\mathbf{r}}{2D} \right) K_{1} \left(\frac{\mathbf{r}}{2D} \right) \right]$$
(14)

In these expressions $C_O(N)$ is the variance of the undulations and $C_O(\Delta g)$ is the variance of the anomalies in the area under consideration with respect to a reference field which is the GEM 9 field in our case. I_n is the Bessel function of the first kind of order n, and K_n is the Bessel function of the second kind of order n. (Although these functions are not defined at r=0, definition is possible at a value of r sufficiently small to approximate zero.) The value of r is the distance between the points under consideration and D is known as the characteristic distance. The values of $C_O(N)$ and $C_O(\Delta g)$ are related as follows:

$$\frac{\sqrt{C_{o}(N)}}{\sqrt{3} D} = \frac{\sqrt{C_{o}(\Delta g)}}{\sqrt{2} g_{o}}$$
 (15)

where go is an average value of gravity (979.8 gals).

We need to develop a procedure to determine the two independent parameters of this model. To do this we use the fact that the mean anomaly (or mean undulation) variance can be computed knowing the point covariance function (Heiskanen and Moritz, 1967, p. 276). Thus, if we subdivide a given area into n² subdivisions we can write

$$var(\overline{\Delta g}) = \frac{1}{n^4} \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{j=1}^{n} C((\Delta g, \Delta g) x_i, y_j, x_i^T, y_j^T)$$
 (16)

where x,y are coordinates within the block. A similar equation can be written for $var(\overline{N})$.

Now suppose we take an area for which we known $\operatorname{var}(\Delta g)$ and $\operatorname{var}(N)$ with respect to some reference field. Then we can use these values to determine $C_0(N)$ and $C_0(\Delta g)$ by solving (16) numerically. Given these two values the characteristic distance D can be found from (15). The process is an iterative one since some starting D value is needed. In our tests, convergence was obtained in an average of 5 iterations by stopping when the change in D was less than 1 meter.

In our application we chose to find the local parameters $C_0(\Delta g)$, and D that would be characteristic of the 1°x 1° anomalies and undulations in specific 5° equal area blocks. The data used for the computation of $var(\overline{\Delta g})$ and $var(\overline{N})$

was the anomaly and undulations predictions with respect to the GEM 9 field when the predictions were done with the covariances from COVA using the T/R anomaly degree variance model. Given these two values we then computed $C_0(\Delta g)$, D and $C_0(N)$ which were then used to generate the covariances from equations (12), (13), and (14).

We selected four 5° equal area blocks to work with. These blocks are numbered (see Figure 4) as follows: 1058 (a block stradling the 90° East Ridge), 1060 (a block in the Indian Ocean with mild anomaly variation with medium altimeter coverage), 602 (a block that contains the Puerto Rican Trench and has dense altimeter coverage), and block 465 (off the east coast of the United States with dense altimeter coverage).

The values of $\sqrt{C_O(N)} = \sigma_N$, $\sqrt{C_O(\Delta g)} = \sigma_g$, D, and the anomaly correlation distance (i.e. the distance at which $C(r) = C_O(\Delta g)/e^{-1}$ (1/e = .3678...)). These values (except for D) are also given, for comparison purposes, for the COVA covariances with the T/R anomaly degree variance model, and the 1L model of Jekeli.

| Block/ Function | $\sigma_{ m g}$ mgals | σ _N meters | D meters | Anomaly Correl. Dist. (km) |
|---|---|--|--|-------------------------------------|
| 1058 1060 602 465 COVA 1 L | 16.09 11.34 121.88 26.55 38.98 31.97 | 1.28 3.00 7.11 2.10 3.64 3.62 | 64042 212365 46841 63637 - | 88 289 64 87 199 154 |

Table 10. Parameters of Covariance Functions.

The large $\sigma_{\rm g}$ value for block 602 reflects the very large (about -250 mgal) anomalies associated with the Puerto Rican Trench.

For blocks 1058, 1060, and 602 predictions were carried out with the three different covariance functions. For block 465 only the COVA with T/R model and the Jordan model were used.

The predicted anomalies were then compared to corresponding terrestrial estimates to see if one covariance function gave better predictions than another. The root mean square (RMS) differences, the RMS terrestrial anomaly standard deviation, and the RMS altimeter derived anomaly standard deviation are given in Table 11.

Table 11. Comparison of 1°x 1° Anomalies Derived Using Different Covariance Functions to Terrestrial Estimates.

| | | • • | | | |
|---|--|----------------------------------|--|--|--|
| Block Number 1058; RMS Terrestrial Anomaly S.D. = $\pm 10.8 \mathrm{mgals}$ | | | | | |
| Covariance | RMS Difference (mgals) | RMS Alt. Anomaly S.D. (mgals) | | | |
| COVA Jekeli 1 L Jordan | ±12.7 12.7 12.0 | ±7.6 6.5 4.8 | | | |
| Block Number 1060; RMS Terrestrial Anomaly S.D. = ± 10.6 mgals | | | | | |
| Covariance | RMS Difference (mgals) | RMS Alt. Anomaly S.D. (mgals) | | | |
| COVA Jekeli 1 L Jordan | ±11.9 11.9 11.9 | ±6.8 5.6 2.4 | | | |
| Block Number 602; RMS Terrestrial Anomaly S.D. = ±11.8 mgals | | | | | |
| Covariance | RMS Difference (mgals) | RMS Alt. Anomaly S.D. (mgals) | | | |
| COVA Jekeli 1 L Jordan | ±12.2 12.4 12.1 | ±6.3 5.1 9.7 | | | |
| Block Number 465; RMS Terrestrial Anomaly S.D. = ±13.8 mgals | | | | | |
| Covariance | RMS RMS Alt. Anomaly Difference (mgals) S.D. (mgals) | | | | |
| COVA Jordan | ±9.0 9.6 | ±6.4 4.7 | | | |

From Table 11 we see that no significant improvement or difference in the prediction results can be seen from the use of different covariance functions. The most significant change occurs in the predicted accuracy of the anomalies where the Jordan function displays accuracies more related to the variations of the anomaly field in a specific area.

We have also compared the anomalies from the Jekeli 1L function and the Jordan function to the anomalies from the COVA 1L function. These comparisons are shown in Table 12.

Table 12. Comparison of the 1°x 1° Anomalies Derived from Two Alternate Covariance Functions to the Values Obtained from the Covariances of COVA.

| Block | Covariance | Mean Diff. | RMS Diff. |
|-------|------------|------------|-----------|
| | Used | (mgals) | (mgals) |
| 1058 | Jekeli 1L | 0.0 | 0.3 |
| 1058 | Jordan | 0.1 | 2.3 |
| 1060 | Jekeli 1L | 0.0 | 0.1 |
| 1060 | Jordan | 0.7 | 3.0 |
| 602 | Jekeli 1L | 0.1 | 0.4 |
| 602 | Jordan | -2.6 | 7.9 |
| 465 | Jordan | -0.3 | 1.5 |

We see that the comparison with the Jekeli 1L function shows differences on the order of 0.3 mgals while the differences with the Jordan function are higher, being about ±3 mgals in three blocks rising to 8 mgals in block 602. The large differences in this latter block occur for anomalies on the order of -280 mgals, the largest difference being 20 mgals. Five specific 1°x 1° anomalies and undulations for blocks along the trench are shown in Table 13 as computed using the three different covariance functions previously discussed.

Table 13. 1° x 1° Anomalies and Undulations in the Area of the Puerto Rican Trench.

| North Cor | | | | Undulations (meters) | | | | |
|----------------------------------|---------------------------------|--|--|--|---|----------------------------------|--|--|
| φ° | λ° | Terr. | COVA | 1L | Jordan | COVA | 1L | Jordan |
| 20 20 20 20 20 20 | 292 293 294 295 296 | -244 ± 24 -282 ± 14 -205 ± 22 -166 ± 10 -162 ± 6 | -264 ± 6 -223 ± 7 -160 ± 6 | -263 ± 6 -222 ± 6 -159 ± 5 | -284 ± 10 -239 ± 12 -172 ± 10 | -65.6±.2 -63.5±.2 -59.7±.2 | -62.3±.2 -65.6±.2 -63.5±.2 -59.7±.2 -59.1±.2 | $-66.1\pm.4$ $-64.0\pm.4$ $-60.0\pm.4$ |

In this section we have examined the variability of the prediction process as a function of the covariances used. There are some indications that the use of a tailored covariance function may give slightly better predictions than global functions but the evidence is marginal. On the other hand the standard deviations from the use of the Jordan function may be more realistic. In some cases these standard deviations are more and in some less than obtained from the global functions. In order to apply the local function we must have representative values for the anomaly and undulation variances. In our application this was simple as predictions had already been carried out. If such predictions had not been done we could have used

the terrestrial data to obtain an anomaly variance and the results of some detailed gooid computations to obtain the undulation variance. If we were working in gravimetrically unsurveyed areas (such as the southern oceans) we could select a global function.

For our production estimation process we choose to work with the covariances implied by COVA with the T/R anomaly degree variance model. This was primarily done for consisting purposes as there did not seem to be sufficient evidence to suggest that significant gains would be obtained from using a tailored covariance function. Perhaps, with more accurate determination of oceanic mean anomalies from terrestrial data, a more definitive comparison and conclusions could be made.

Production Estimation of Mean Anomalies and Undulations

The estimation of 1°x 1° and 5° mean anomalies (and undulation) using equation (7) took place using the old adjustment where data was sufficiently dense, and using the new adjustment data in other areas. Many of the old adjustment anomalies were taken from Rapp (1977a). The covariance function used was obtained from COVA (Tscherning and Rapp, 1974) with respect to a degree 20 field. Other specific details of the prediction process are described in Rapp (1977a). The values from the old and the new adjustment were merged together to form a combined data set. This data set contained 29479 1° x 1° blocks and 957 5° equal area blocks. A number of the $1^{\circ}x$ 1° predictions were made in land areas and in ocean areas where the altimeter data was sparse. This was done only because of the manner chosen for the estimation of all the 1°x 1° anomalies within a 5° equal area block. The more reliable anomalies are those having an accuracy of ±15 mgals or better. There are 27466 such values whose location is shown in Figure 11. (As a matter of interest Figure 12 shows the 20599 values where the accuracy is ± 8 mgals or better.) A listing of the 5° equal area anomalies and undulations, referred to an ellipsoid whose flattening is 1/298.256, is given in the appendix. A tape containing the 1°x 1° altimeter derived anomalies is also available. The predicted anomalies have been compared to a terrestrial data set called "June 78 delete 424". This data set is that terrestrial field described in Rapp (1978b) less 424 anomalies that had a very large difference with the altimeter values. The net data set available for comparison purposes contained 38981 values. This data set was also used to generate a 5° equal area anomaly field that was used for the 5° block comparisons. The 5° comparisons were made using only those 5° terrestrial anomalies where the terrestrial standard deviation was 10 mgals or less. The 1° x 1° comparisons were made only when the accuracy estimates for both anomalies were 15 mgals or better. The results are given in Table 14.



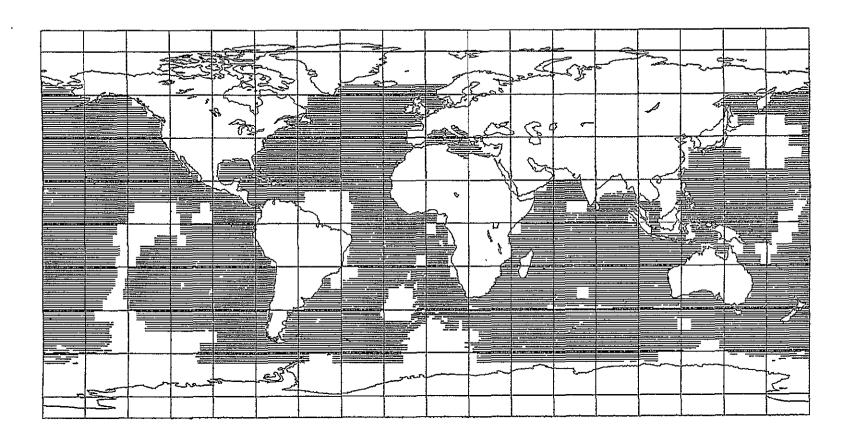


Figure 11. Location of 27466 1° x 1° Anomalies Derived from Geos-3 Altimeter Data Where the Accuracy is ± 15 mgals or Better.

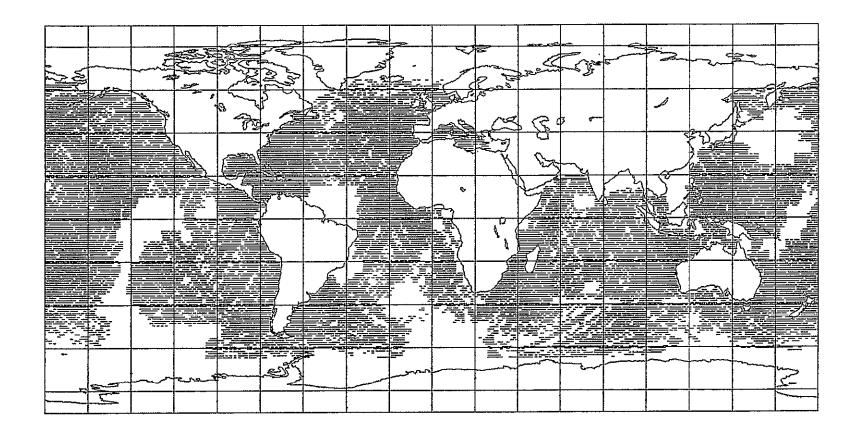


Figure 12. Location of 20599 $1^{\circ} \times 1^{\circ}$ Anomalies Derived from Geos-3 Altimeter Data where the Accuracy is ± 8 mgals or Better.

Table 14. Comparison of 5° Equal Area, and 1°x 1°, Altimeter Derived and Terrestrial Mean Free Air Anomalies.

| Description | 5° EA Value | 1° x 1° Value |
|-------------------------------------|-------------|---------------|
| Mean Difference (GEM 9-Terr.) | 0.3 mgals | |
| Mean Difference (Alt Terr.) | 0.7 | 0.5 |
| RMS Difference (GEM 9 - Terr.) | ±8.9 | 23.4 |
| RMS Difference (Alt Terr.) | 6.8 | 11. 8 |
| RMS Terrestrial Accuracy | 4.8 | 10.9 |
| RMS Altimeter Accuracy | 2.7 | 7.8 |
| RMS Terrestrial Anomaly | 15.2 | 27.7 |
| Maximum Difference | 35.9 | 63.8 |
| Number of Differences > y mgals | 10 * | 7† |
| Number of blocks compared | 767 | 10086 |

^{*} y = 20, t y = 40

We see that the altimeter anomalies have a better agreement with the terrestrial anomalies than the GEM 9 anomalies (computed from potential coefficients to degree 20) as would be expected. The average accuracy of the 5° altimeter anomalies is 3 mgals while it is 4 mgals for the terrestrial data used. The RMS difference between the 5° terrestrial and altimeter anomalies of 6.8 mgals is somewhat greater than would be expected if the terrestrial and altimeter accuracies were correct. We finally note a very small (0.7 mgal) systematic difference between the terrestrial and altimeter anomalies.

The 1°x 1° anomaly comparisons show a 11.8 mgals RMS differences between the altimeter and terrestrial data with only a 0.5 mgals systematic difference. This RMS difference is somewhat smaller than would be expected from the average accuracy estimates of the two data types which is about 11 mgals for the terrestrial data and 8 mgals for the altimeter values.

The 1°x 1° mean geoid undulations estimated from the altimeter data were compared to the GEM 9 undulations (computed to degree 20) where we found a mean difference of 0.0 meters and a root mean square difference of ± 2.7 m. This difference is consistent with the expected value of ± 3.2 meters.

Estimation in Small Areas

The preceding computations have described the estimation of mean anomalies and mean undulations. It is of interest to examine the values to be obtained on the scale smaller than the $1^{\circ} \times 1^{\circ}$ anomalies considered in this report. One attempt at this is described in Rapp (1978a, 1979) where point anomaly profiles were constructed across the Bonin Trench and around a sea mount in the Gulf of Alaska. In the case of the trench we saw an anomaly change of 443 mgals in

118 km. This large change indicated that there was high frequency anomaly information within the altimeter derived undulations. However we did not have at that time ship gravity data to compare with our altimeter derived anomalies.

Recently Detrick and Watts (1979) have described some investigations near the Ninety-East Ridge. These studies pointed out a region of available ship gravity data with bathymetric data that would provide a useful test area.

We first constructed an altimeter gooid in the area of the ridge which is shown as Figure 13. This map shows the undulation contours at 1 m intervals with the altimeter tracks plotted. In addition the 4000 m depth contour taken from Sclater and Fisher (1974) has been plotted to aide in the identification of the ridge crest. The depth of the actual crest varies approximately between 2000 m and 3500 meters.

This map has been prepared using the altimeter data of the first adjustment using a prediction interval of 0.5×0.5 . In addition a few bad data points (newly discovered) were removed from the data set. Consequently this geoid will differ somewhat from that global representation described earlier.

For much of the geoid map there seems to be no specific association with the ridge although some dependence may be seen in the more southern parts.

To specifically test the anomaly prediction process ship data was obtained from Watts (private communication). The tracks obtained corresponded to some of the profiles described in Detrick and Watts (1979).

The point anomalies were predicted from the altimeter data at the same points as the existing ship gravity measurements. The prediction was carried out using one or two data selections and matrix inversions per profile. The results of these predictions are shown in Figures 14 and 15 where we have plotted the altimeter derived geoid, the altimeter derived anomalies, the ship determined anomalies and the measured bathymetry.

The profile shown in Figure 14 corresponds to the data used to obtain profile 90 E-9 in Detrick and Watts (1979). The starting (S) and ending (E) coordinates are: $\varphi_S = -3^{\circ}.19$, $\lambda_S = 85^{\circ}.115$, $\varphi_E = -4^{\circ}.00$, $\lambda_E = 93^{\circ}.515$. The profile shown in Figure 15 corresponds to the data used to obtain profile 90 E-12 in Detrick and Watts (1979). The starting and ending coordinates are: $\varphi_S = -16^{\circ}.755$ $\lambda_S = 8^{\circ}.3225$, $\varphi_E = -17^{\circ}.507$, $\lambda_E = 91^{\circ}.874$.

We can see that the altimeter anomalies follow quite closely the ship data except for the high frequency components. There is a clear correlation of the altimeter derived anomalies with the bathymetry. On the other hand the geoid seems to vary quite smoothly across the ridge, showing a small bump at the ridge crest.

Figure 13.

Altimeter Geoid in the Area of the Ninety-East Ridge Showing Location of 4000m Depth and Altimeter Tracks. Contour Interval = 1 m.

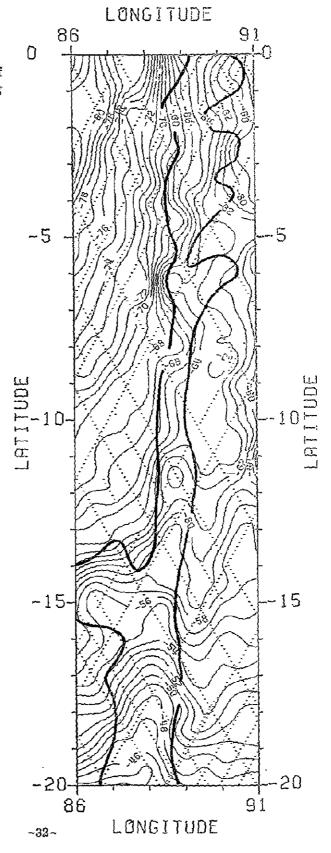


FIGURE 14. UNDULATION, ANOMALY, AND BATHYMETRY ACROSS 90 EAST RIDGE AT $\phi \sim -3.6^{\circ}$

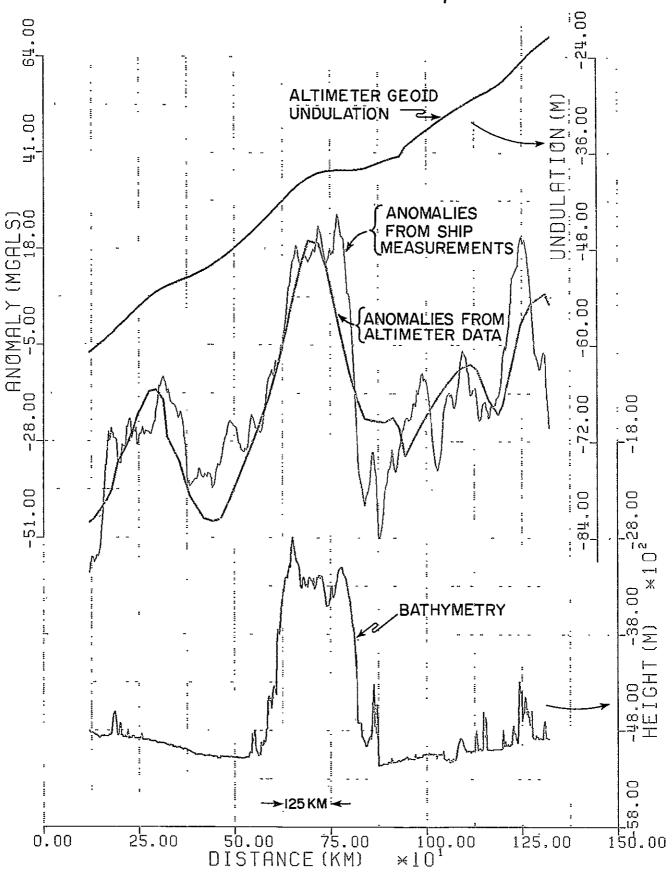
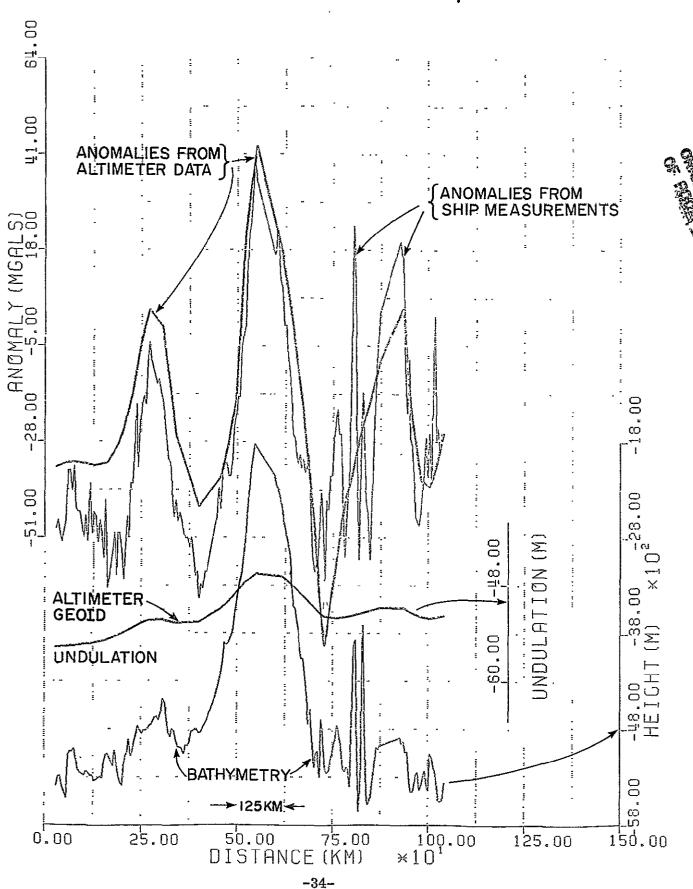


FIGURE 15. UNDULATION, ANOMALY, AND BATHYMETRY ACROSS 90 EAST RIDGE AT $\phi \sim -17^{\circ}$



The RMS differences between the ship data and the altimeter data for the profile shown in Figures 14 and 15 was 12 mgals and 20 mgals respectively. The average predicted accuracy was 26 mgals. If the ship data is filtered to remove the high frequency components this agreement between the ship and the altimeter derived anomalies improves. For example, Eren (1979, private communication) has shown that if the components below a wavelength of 100 km are filtered out, the RMS differences for both profiles shown in Figure 14 and 15 become ± 11 mgals.

There seems to be two conclusions to be reached from these specific studies. First the altimeter derived anomalies agree quite well with the ship data considering the accuracy of both data types. The altimeter anomalies seem to reflect the general bathymetry but they do not reflect the high frequency information seen by the ship measurements. More detailed analysis is needed to assess the accuracy of both data types by wavelength. In addition we need to look at the use of more detailed altimeter data as opposed to using from averages which represent an immediate averaging and loss of high frequency information.

Second we see that the geoid undulations only slightly reflect the variations going over the ridge. The significant variation is seen much more in examining the anomaly data.

Summary and Conclusions

This report has described the processing and analysis of Geos-3 data released for general use by March 1978. The data was edited and adjusted to remove bad data and to remove orbit and altimeter bias terms. The adjustment was carried out first in a primary set involving 700 arcs chosen for their global distribution, and 4 regional adjustments. The crossover discrepancies after the adjustment averaged about ±55 cm.

The resultant data was used to prepare a global sea surface map (or approximately the geoid) with accuracy estimates. Comparisons of these maps with similar data produced using precise orbits indicated random differences on the order of 1 meter with a systematic difference implying an equatorial radius of 6378137 meters. Computations were also done to investigate the accuracy of the altimeter geoid by wavelength. Comparisons with the undulation implied by the GEM 9 potential coefficients showed differences of about 75 cm at 5 wavelengths from 13010 km to 2602 km.

Several investigations were also carried out to improve our mean anomaly and mean undulation computation procedures. Specifically we considered the use of the smoothing operator in the series covariance expressions instead of numerical integration procedures for mean covariance computations. Although the procedure could save some computer time without any significant loss of accuracy, the original numerical integration procedure was retained for logistical reasons.

We also considered the correlation between various $1^{\circ} \times 1^{\circ}$ predictions within a 5° equal area block. We found that the average correlation coefficient between adjacent blocks was about 0.2 so that the blocks could be considered to have been determined independently.

The altimeter data was then used to extend our 5° and $1^{\circ}x$ 1° mean anomaly and undulation computation. For some unknown reason the data from our original adjustment in 1977 gave somewhat better anomalies than the new adjustment. Consequently in those geographic areas where the old adjustment data was adequate we used that data for anomaly estimation. In the other areas the new adjustment data was used. A total of 29479 $1^{\circ}x$ 1° blocks and 957 5° equal area blocks were estimated. Of these $1^{\circ}x$ 1° values 27466 had predicted standard deviations of ± 15 mgals or better. Comparisons were made with the terrestrial anomaly data where we found differences on the order of ± 12 mgals for the $1^{\circ}x$ 1° data and ± 7 mgals for the 5° data. A good part of this difference is due to the errors in the terrestrial anomaly field.

The anomaly and undulation predictions were done with a global covariance function referred to a degree 20 reference field. Additional tests were carried out with a different global covariance function and a local covariance function derived for special areas. No significant changes were seen in the predictions from the two global field models. Root mean square differences on the order of 3 mgals were found between the global and local covariance function but no significant improvement in the prediction process was seen based on comparisons with actual data. We did see changes in the predicted standard deviations up to 65% when using the local model instead of the global covariance model. The difficulty in applying a local model lies in a need for an adequate knowledge of the residual anomaly and undulation field in the area.

Point anomaly and undulation values were also computed along ship tracks that crossed the Ninety-East Ridge. The resultant anomaly values were compared to ship data where root mean square differences were 12 and 20 mgals for the two tracks considered. Plots of this data with the bathymetric indicate the correlation of the altimeter anomalies with the bathymetry on a regional basis as opposed to a more local correlation seen in the ship gravity data. Part of this difference may be due to our use of altimeter averages over 14 to 20 km swaths. However, we clearly can see the potential for improved knowledge of the anomaly field in small areas using the altimeter data.

What additional things need to be done? First the data needs to be reexamined to try to delete some additional bad data that has shown up. We should try to get additional data to fill in those areas for which data is non-existent or sparse. Then a readjustment could be done and the anomalies and undulations computed using localized covariance functions. One should also examine the use of non-frame averages going back, perhaps to the original data, or 1 second averages. The information, by wavelength, in the altimeter data should be studied to examine the accuracy of our anomaly and undulation results. And, of course, attempts have been made, and could continue to define sea surface topography effects.

Finally, it sould be clear that this Geos-3 altimeter data has enabled a significant improvement in our knowledge of the earth's gravity field. It's use in local areas and in global computations has added significantly to geodetic and geophysical research. This data and Seasat-1 data will provide a data source for additional research into the gravity field at sea and indirectly on a global basis.

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Appendix

This appendix contains a listing of the 5° equal area free-air anomalies and undulations as derived from Geos-3 altimeter data. These values refer to an ellipsoid whose flattening is 1/298.257 and whose equatorial radius is theoretically unknown, being actually the equatorial radius of the general terrestrial ellipsoid.

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